Chapter 9 Topics

9-1 Reflections
  Reflect in the...
  x-axis (a, b) →
  y-axis (a, b) →
  origin (a, b) →
  y = x (a, b) →
  Line of symmetry:
  Point of symmetry:

9-2 Translation
  (x, y) →
  Composition of reflections in 2 parallel lines

9-3 Rotations
  Order
  Magnitude
  Composition of reflections in 2 intersecting lines
  The angle of rotation is twice the measure of the acute angle formed by the intersection
  Rotate clockwise 90° about a point (p,q): (x,y) →
  Rotate counterclockwise 90° about a point (p,q): (x, y) →

9-4 Tessellations
  Regular tessellations
  Semi-regular tessellations
  Uniform

9-5 Dilations
  A'B' = |r|AB
  Draw Dilations

9-6 Vectors
  Component <change in x, change in y>
  direction
  magnitude

9-7 Matrices
  Translate, dilate, reflect, and rotate
  Use Vertex matrix and transformation matrix to find image coordinates.
  Make sure dimensions are correct!
Draw Reflections The transformation called a reflection is a flip of a figure in a point, a line, or a plane. The new figure is the image and the original figure is the preimage. The preimage and image are congruent, so a reflection is a congruence transformation or isometry.

Example 1 Construct the image of quadrilateral $ABCD$ under a reflection in line $m$.

![Diagram of quadrilateral ABCD with reflected image A'B'C'D']

Draw a perpendicular from each vertex of the quadrilateral to $m$. Find vertices $A'$, $B'$, $C'$, and $D'$ that are the same distance from $m$ on the other side of $m$. The image is $A'B'C'D'$.

Example 2 Quadrilateral $DEFG$ has vertices $D(-2, 3)$, $E(4, 4)$, $F(3, -2)$, and $G(-3, -1)$. Find the image under reflection in the $x$-axis.

To find an image for a reflection in the $x$-axis, use the same $x$-coordinate and multiply the $y$-coordinate by $-1$. In symbols, $(a, b) \rightarrow (a, -b)$. The new coordinates are $D'(-2, -3)$, $E'(4, -4)$, $F'(3, 2)$, and $G'(-3, 1)$. The image is $D'E'F'G'$.

In Example 2, the notation $(a, b) \rightarrow (a, -b)$ represents a reflection in the $x$-axis. Here are three other common reflections in the coordinate plane.

- in the $y$-axis: $(a, b) \rightarrow (-a, b)$
- in the line $y = x$: $(a, b) \rightarrow (b, a)$
- in the origin: $(a, b) \rightarrow (-a, -b)$

Exercises

Draw the image of each figure under a reflection in line $m$.

1. 

![Diagram of figure KJH with reflected image K'J'H']

2. 

![Diagram of figure LMN with reflected image L'M'N']

3. 

![Diagram of figure RSTU with reflected image R'S'T'U']

Graph each figure and its image under the given reflection.

4. Triangle $DEF$ with $D(-2, -1)$, $E(-1, 3)$, $F(3, -1)$ in the $x$-axis

![Diagram of triangle DEF with reflected image D'E'F']

5. Quadrilateral $ABCD$ with $A(1, 4)$, $B(3, 2)$, $C(2, -2)$, $D(-3, 1)$ in the $y$-axis

![Diagram of quadrilateral ABCD with reflected image A'B'C'D']
Lines and Points of Symmetry If a figure has a line of symmetry, then it can be folded along that line so that the two halves match. If a figure has a point of symmetry, it is the midpoint of all segments between the preimage and image points.

Example Determine how many lines of symmetry a regular hexagon has. Then determine whether a regular hexagon has point symmetry.
There are six lines of symmetry, three that are diagonals through opposite vertices and three that are perpendicular bisectors of opposite sides. The hexagon has point symmetry because any line through \( P \) identifies two points on the hexagon that can be considered images of each other.

Exercises Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

1.  
   ![Diagram of a rhombus]

2.  
   ![Diagram of an equilateral triangle]

3.  
   ![Diagram of a rectangle]

4.  
   ![Diagram of a pentagon]

5.  
   ![Diagram of a parallelogram]

6.  
   ![Diagram of a triangle]

7.  
   ![Diagram of a trapezoid]

8.  
   ![Diagram of a parallelogram]

9.  
   ![Diagram of a quadrilateral]
9-2 Study Guide and Intervention

Translations

Translations Using Coordinates A transformation called a translation slides a figure in a given direction. In the coordinate plane, a translation moves every preimage point \( P(x, y) \) to an image point \( P(x + a, y + b) \) for fixed values \( a \) and \( b \). In words, a translation shifts a figure \( a \) units horizontally and \( b \) units vertically; in symbols, \( (x, y) \rightarrow (x + a, y + b) \).

Example Rectangle \( RECT \) has vertices \( R(-2, -1) \), \( E(-2, 2) \), \( C(3, 2) \), and \( T(3, -1) \). Graph \( RECT \) and its image for the translation \( (x, y) \rightarrow (x + 2, y - 1) \).

The translation moves every point of the preimage right 2 units and down 1 unit.

\[
(x, y) \rightarrow (x + 2, y - 1)
\]

- \( R(-2, -1) \rightarrow R'(0, -2) \)
- \( E(-2, 2) \rightarrow E'(0, 1) \)
- \( C(3, 2) \rightarrow C'(5, 1) \)
- \( T(3, -1) \rightarrow T'(5, -2) \)

Exercises

Graph each figure and its image under the given translation.

1. \( \overline{PQ} \) with endpoints \( P(-1, 3) \) and \( Q(2, 2) \) under the translation left 2 units and up 1 unit

2. \( \triangle PQR \) with vertices \( P(-2, -4) \), \( Q(-1, 2) \), and \( R(2, 1) \) under the translation right 2 units and down 2 units

3. Square \( SQUR \) with vertices \( S(0, 2) \), \( Q(3, 1) \), \( U(2, -2) \), and \( R(-1, -1) \) under the translation right 3 units and up 1 unit
Translations by Repeated Reflections Another way to find the image of a translation is to reflect the figure twice in parallel lines. This kind of translation is called a composite of reflections.

**Example** In the figure, \( m \parallel n \). Find the translation image of \( \triangle ABC \).
\( \triangle A'B'C' \) is the image of \( \triangle ABC \) reflected in line \( m \).
\( \triangle A''B''C'' \) is the image of \( \triangle A'B'C' \) reflected in line \( n \).
The final image, \( \triangle A''B''C'' \), is a translation of \( \triangle ABC \).

**Exercises**

In each figure, \( m \parallel n \). Find the translation image of each figure by reflecting it in line \( m \) and then in line \( n \).

1. 

2. 

3. 

4. 

5. 

6. 
Skills Practice
Translations
In each figure, $a \parallel b$. Determine whether figure 3 is a translation image of figure 1. Write yes or no. Explain your answer.

1. 

2. 

3. 

4. 

COORDINATE GEOMETRY Graph each figure and its image under the given translation.

5. $\triangle JKL$ with vertices $J(-4, -4)$, $K(-2, -1)$, and $L(2, -4)$ under the translation $(x, y) \rightarrow (x + 2, y + 5)$

6. quadrilateral $LMNP$ with vertices $L(4, 2)$, $M(4, -1)$, $N(0, -1)$, and $P(1, 4)$ under the translation $(x, y) \rightarrow (x - 4, y - 3)$
9-2 Practice
Translations

In each figure, $c \parallel d$. Determine whether figure 3 is a translation image of figure 1. Write yes or no. Explain your answer.

1.

2.

COORDINATE GEOMETRY Graph each figure and its image under the given translation.

3. quadrilateral $TUWX$ with vertices $T(-1, 1), U(4, 2), W(1, 5)$, and $X(-1, 3)$ under the translation $(x, y) \rightarrow (x - 2, y - 4)$

4. pentagon $DEFGH$ with vertices $D(-1, -2), E(2, -1), F(5, -2), G(4, -4), H(1, -4)$ under the translation $(x, y) \rightarrow (x - 1, y + 5)$

ANIMATION Find the translation that moves the figure on the coordinate plane.

5. figure 1 $\rightarrow$ figure 2

6. figure 2 $\rightarrow$ figure 3

7. figure 3 $\rightarrow$ figure 4
### Study Guide and Intervention

#### Rotations

**Draw Rotations** A transformation called a rotation turns a figure through a specified angle about a fixed point called the center of rotation. To find the image of a rotation, one way is to use a protractor. Another way is to reflect a figure twice, in two intersecting lines.

**Example 1** \( \triangle ABC \) has vertices \( A(2, 1), B(3, 4), \) and \( C(5, 1) \). Draw the image of \( \triangle ABC \) under a rotation of 90° counterclockwise about the origin.

- First draw \( \triangle ABC \). Then draw a segment from \( O \), the origin, to point \( A \).
- Use a protractor to measure 90° counterclockwise with \( \overline{OA} \) as one side.
- Draw \( \overline{OR} \).
- Use a compass to copy \( \overline{OA} \) onto \( \overline{OR} \). Name the segment \( \overline{OA'} \).
- Repeat with segments from the origin to points \( B \) and \( C \).

**Example 2** Find the image of \( \triangle ABC \) under reflection in lines \( m \) and \( n \).

First reflect \( \triangle ABC \) in line \( m \). Label the image \( \triangle A'B'C' \).

Reflect \( \triangle A'B'C' \) in line \( n \). Label the image \( \triangle A''B''C'' \).

\( \triangle A''B''C'' \) is a rotation of \( \triangle ABC \). The center of rotation is the intersection of lines \( m \) and \( n \). The angle of rotation is twice the measure of the acute angle formed by \( m \) and \( n \).

**Exercises**

Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.

1. \( \overline{PQ} \) with endpoints \( P(-1, -2) \) and \( Q(1, 3) \) counterclockwise about the origin

2. \( \triangle PQR \) with vertices \( P(-2, -3) \), \( Q(2, -1) \), and \( R(3, 2) \) clockwise about the point \( T(1, 1) \)

Find the rotation image of each figure by reflecting it in line \( m \) and then in line \( n \).

3.

4.
Rotations

Rotational Symmetry. When the figure at the right is rotated about point \( P \) by 120° or 240°, the image looks like the preimage. The figure has rotational symmetry, which means it can be rotated less than 360° about a point and the preimage and image appear to be the same.

The figure has rotational symmetry of order 3 because there are 3 rotations less than 360° (0°, 120°, 240°) that produce an image that is the same as the original. The magnitude of the rotational symmetry for a figure is 360 degrees divided by the order. For the figure above, the rotational symmetry has magnitude 120 degrees.

Example Identify the order and magnitude of the rotational symmetry of the design at the right.

The design has rotational symmetry about the center point for rotations of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°.

There are eight rotations less than 360 degrees, so the order of its rotational symmetry is 8. The quotient \( 360 \div 8 \) is 45, so the magnitude of its rotational symmetry is 45 degrees.

Exercises

Identify the order and magnitude of the rotational symmetry of each figure.

1. a square
2. a regular 40-gon

3.

4.

5.

6.
Skills Practice
Rotations

Rotate each figure about point $R$ under the given angle of rotation and the given direction. Label the vertices of the rotation image.

1. $90^\circ$ counterclockwise

2. $90^\circ$ clockwise

COORDINATE GEOMETRY Draw the rotation image of each figure $90^\circ$ in the given direction about the origin and label the coordinates.

3. $\triangle STW$ with vertices $S(2, -1)$, $T(5, 1)$, and $W(3, 3)$ counterclockwise

4. $\triangle DEF$ with vertices $D(-4, 3)$, $E(1, 2)$, and $F(-3, -3)$ clockwise

Use a composition of reflections to find the rotation image with respect to lines $\hat{k}$ and $m$. Then find the angle of rotation for each image.

5.

6.
9-3 Practice

Rotations

Rotate each figure about point \( R \) under the given angle of rotation and the given direction. Label the vertices of the rotation image.

1. \( 80^\circ \) counterclockwise

\[
\begin{array}{c}
\text{N} \\
\text{M} \\
\text{P}
\end{array}
\]

2. \( 100^\circ \) clockwise

\[
\begin{array}{c}
\text{P} \\
\text{Q} \\
\text{T} \\
\text{S}
\end{array}
\]

COORDINATE GEOMETRY Draw the rotation image of each figure \( 90^\circ \) in the given direction about the center point and label the coordinates.

3. \( \triangle RST \) with vertices \( R(-3, 3), S(2, 4), \) and \( T(1, 2) \) clockwise about the point \( P(1, 0) \)

4. \( \triangle HJK \) with vertices \( H(3, 1), J(3, -3), \) and \( K(-3, -4) \) counterclockwise about the point \( P(-1, -1) \)

Use a composition of reflections to find the rotation image with respect to lines \( p \) and \( s \). Then find the angle of rotation for each image.

5. 

6. 

7. STEAMBOATS A paddle wheel on a steamboat is driven by a steam engine and moves from one paddle to the next to propel the boat through the water. If a paddle wheel consists of 18 evenly spaced paddles, identify the order and magnitude of its rotational symmetry.
**Study Guide and Intervention**

**Tessellations**

**Regular Tessellations** A pattern that covers a plane with repeating copies of one or more figures so that there are no overlapping or empty spaces is a **tessellation**. A **regular tessellation** uses only one type of regular polygon. In a tessellation, the sum of the measures of the angles of the polygons surrounding a vertex is 360. If a regular polygon has an interior angle that is a factor of 360, then the polygon will tessellate.

![Regular Tessellation]

**Example** Determine whether a regular 16-gon tessellates the plane. Explain.

If \( m\angle 1 \) is the measure of one interior angle of a regular polygon, then a formula for \( m\angle 1 \) is \( m\angle 1 = \frac{180(n - 2)}{n} \). Use the formula with \( n = 16 \).

\[
m\angle 1 = \frac{180(n - 2)}{n} = \frac{180(16 - 2)}{16} = 157.5
\]

The value 157.5 is not a factor of 360, so the 16-gon will not tessellate.

**Exercises**

Determine whether each polygon tessellates the plane. If so, draw a sample figure.

1. scalene right triangle
2. isosceles trapezoid

Determine whether each regular polygon tessellates the plane. Explain.

3. square
4. 20-gon
5. septagon
6. 15-gon
7. octagon
8. pentagon
Tessellations

Tessellations with Specific Attributes A tessellation pattern can contain any type of polygon. If the arrangement of shapes and angles at each vertex in the tessellation is the same, the tessellation is uniform. A semi-regular tessellation is a uniform tessellation that contains two or more regular polygons.

Example Determine whether a kite will tessellate the plane. If so, describe the tessellation as uniform, regular, semi-regular, or not uniform. A kite will tessellate the plane. At each vertex the sum of the measures is \(a + b + b + c\), which is 360. The tessellation is uniform.

Exercises

Determine whether a semi-regular tessellation can be created from each set of figures. If so, sketch the tessellation. Assume that each figure has a side length of 1 unit.

1. rhombus, equilateral triangle, and octagon
2. square and equilateral triangle

Determine whether each polygon tessellates the plane. If so, describe the tessellation as uniform, not uniform, regular, or semi-regular.

3. rectangle

4. hexagon and square
9-4 Skills Practice

Tessellations

Determine whether each regular polygon tessellates the plane. Explain.

1. 15-gon
2. 18-gon
3. square
4. 20-gon

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

5. regular pentagons and equilateral triangles
6. regular dodecagons and equilateral triangles
7. regular octagons and equilateral triangles

Determine whether each polygon tessellates the plane. If so, describe the tessellation as uniform, not uniform, regular, or semi-regular.

8. rhombus
9. isosceles trapezoid and square

Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.

10.

11.
9-4

Practice

Tessellations

Determine whether each regular polygon tessellates the plane. Explain.

1. 22-gon
2. 40-gon

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

3. regular pentagons and regular decagons

4. regular dodecagons, regular hexagons, and squares

Determine whether each polygon tessellates the plane. If so, describe the tessellation as uniform, not uniform, regular, or semi-regular.

5. kite

6. octagon and decagon

Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.

7. 

8. 

FLOOR TILES For Exercises 9 and 10, use the following information.

Mr. Martinez chose the pattern of tile shown to retile his kitchen floor.

9. Determine whether the pattern is a tessellation. Explain

10. Is the pattern uniform, regular, or semi-regular?
9-5 Study Guide and Intervention

Dilations

Classify Dilations  A dilation is a transformation in which the image may be a different size than the preimage. A dilation requires a center point and a scale factor, r.

Let $r$ represent the scale factor of a dilation.
If $|r| > 1$, then the dilation is an enlargement.
If $|r| = 1$, then the dilation is a congruence transformation.
If $0 < |r| < 1$, then the dilation is a reduction.

**Example**

Draw the dilation image of $\triangle ABC$ with center $O$ and $r = 2$.

Draw $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$. Label points $A'$, $B'$, and $C'$ so that $OA' = 2(OA)$, $OB' = 2(OB)$, and $OC' = 2(OC)$. $\triangle A'B'C'$ is a dilation of $\triangle ABC$.

**Exercises**

Draw the dilation image of each figure with center $C$ and the given scale factor. Describe each transformation as an enlargement, congruence, or reduction.

1. $r = 2$

2. $r = \frac{1}{2}$

3. $r = 1$

4. $r = 3$

5. $r = \frac{2}{3}$

6. $r = 1$
9-5 Study Guide and Intervention (continued)

Dilations

Identify the Scale Factor  If you know corresponding measurements for a preimage and its dilation image, you can find the scale factor.

Example  Determine the scale factor for the dilation of $XY$ to $AB$. Determine whether the dilation is an enlargement, reduction, or congruence transformation.

scale factor $\frac{\text{image length}}{\text{preimage length}}$  
$= \frac{8 \text{ units}}{4 \text{ units}}$  
$= 2$

The scale factor is greater than 1, so the dilation is an enlargement.

Exercises

Determine the scale factor for each dilation with center $C$. Determine whether the dilation is an enlargement, reduction, or congruence transformation.

1. $CGHJ$ is a dilation image of $CDEF$.

2. $\triangle CKL$ is a dilation image of $\triangle CKL$.

3. $STUVWX$ is a dilation image of $MNOPQR$.

4. $\triangle CPQ$ is a dilation image of $\triangle CYZ$.

5. $\triangle EFG$ is a dilation image of $\triangle ABC$.

6. $\triangle HJK$ is a dilation image of $\triangle HJK$. 
9-5 Skills Practice

Dilations

Draw the dilation image of each figure with center C and the given scale factor.

1. \( r = 2 \)

\[ \triangle \]

\[ C \cdot \]

2. \( r = \frac{1}{4} \)

\[ \square \]

\[ C \cdot \]

Find the measure of the dilation image \( M'N' \) or of the preimage \( MN \) using the given scale factor.

3. \( MN = 3, \, r = 3 \)

4. \( M'N' = 7, \, r = 21 \)

COORDINATE GEOMETRY Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of \( \frac{1}{2} \).

5. \( J(2, 4), K(4, 4), P(3, 2) \)

\[ \text{Grid} \]

6. \( D(-2, 0), G(0, 2), F(2, -2) \)

\[ \text{Grid} \]

Determine the scale factor for each dilation with center C. Determine whether the dilation is an enlargement, reduction, or congruence transformation. The dashed figure is the dilation image.

7.

\[ \text{Grid} \]

8.

\[ \text{Grid} \]
Reading to Learn Mathematics

V

Vectors

Pre-Activity  How do vectors help a pilot plan a flight?

Read the introduction to Lesson 9-6 at the top of page 498 in your textbook.

Why do pilots often head their planes in a slightly different direction from their destination?

Reading the Lesson

1. Supply the missing words or phrases to complete the following sentences.
   a. A __________ is a directed segment representing a quantity that has both magnitude and direction.
   b. The length of a vector is called its __________.
   c. Two vectors are parallel if and only if they have the same or __________ direction.
   d. A vector is in __________ if it is drawn with initial point at the origin.
   e. Two vectors are equal if and only if they have the same __________ and the same __________.
   f. The sum of two vectors is called the __________.
   g. A vector is written in __________ if it is expressed as an ordered pair.
   h. The process of multiplying a vector by a constant is called __________.

2. Write each vector described below in component form.
   a. a vector in standard position with endpoint \((a, b)\)
   b. a vector with initial point \((a, b)\) and endpoint \((c, d)\)
   c. a vector in standard position with endpoint \((-3, 5)\)
   d. a vector with initial point \((2, -3)\) and endpoint \((6, -8)\)
   e. \(\vec{a} + \vec{b}\) if \(\vec{a} = \langle -3, 5 \rangle\) and \(\vec{b} = \langle 6, -4 \rangle\)
   f. \(5\vec{u}\) if \(\vec{u} = \langle 8, -6 \rangle\)
   g. \(-\frac{1}{3}\vec{v}\) if \(\vec{v} = \langle -15, 24 \rangle\)
   h. \(0.5\vec{u} + 1.5\vec{v}\) if \(\vec{u} = \langle 10, -10 \rangle\) and \(\vec{v} = \langle -8, 6 \rangle\)

Helping You Remember

3. A good way to remember a new mathematical term is to relate it to a term you already know. You learned about scale factors when you studied similarity and dilations. How is the idea of a scalar related to scale factors?
Study Guide and Intervention

**Vectors**

**Magnitude and Direction** A vector is a directed segment representing a quantity that has both magnitude, or length, and direction. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as $\overrightarrow{AB}$, where $A$ is the initial point and $B$ is the endpoint, or as $\overrightarrow{v}$.

A vector in **standard position** has its initial point at $(0, 0)$ and can be represented by the ordered pair for point $B$. The vector at the right can be expressed as $\overrightarrow{v} = (5, 3)$.

You can use the Distance Formula to find the magnitude $|\overrightarrow{AB}|$ of a vector. You can describe the direction of a vector by measuring the angle that the vector forms with the positive $x$-axis or with any other horizontal line.

**Example** Find the magnitude and direction of $\overrightarrow{AB}$ for $A(5, 2)$ and $B(8, 7)$.

Find the magnitude.

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8 - 5)^2 + (7 - 2)^2}$$

$$= \sqrt{34}$$ or about 5.8 units

To find the direction, use the tangent ratio.

$$\tan A = \frac{5}{3}$$ The tangent ratio is opposite over adjacent.

$$m\angle A \approx 59.0$$ Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is $59^\circ$.

**Exercises**

Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates. Round to the nearest tenth.

1. $A(3, 1), B(-2, 3)$

2. $A(0, 0), B(-2, 1)$

3. $A(0, 1), B(3, 5)$

4. $A(-2, 2), B(3, 1)$

5. $A(3, 4), B(0, 0)$

6. $A(4, 2), B(0, 3)$
Study Guide and Intervention (continued)

Vectors

Translations with Vectors  Recall that the transformation \((a, b) \rightarrow (a + 2, b - 3)\) represents a translation right 2 units and down 3 units. The vector \((2, -3)\) is another way to describe that translation. Also, two vectors can be added: \((a, b) + (c, d) = (a + c, b + d)\). The sum of two vectors is called the resultant.

Example  Graph the image of parallelogram \(RSTU\) under the translation by the vectors \(\vec{m} = (3, -1)\) and \(\vec{n} = (-2, -4)\).

Find the sum of the vectors.
\[
\vec{m} + \vec{n} = (3, -1) + (-2, -4)
= (3 - 2, -1 - 4)
= (1, -5)
\]

Translate each vertex of parallelogram \(RSTU\) right 1 unit and down 5 units.

Exercises

Graph the image of each figure under a translation by the given vector(s).

1. \(\triangle ABC\) with vertices \(A(-1, 2), B(0, 0),\) and \(C(2, 3); \vec{m} = (2, -3)\)

2. \(ABCD\) with vertices \(A(-4, 1), B(-2, 3), C(1, 1),\) and \(D(-1, -1); \vec{n} = (3, -3)\)

3. \(ABCD\) with vertices \(A(-3, 3), B(1, 3), C(1, 1),\) and \(D(-3, 1);\) the sum of \(\vec{p} = (-2, 1)\) and \(\vec{q} = (5, -4)\)

Given \(\vec{m} = (1, -2)\) and \(\vec{n} = (-3, -4)\), represent each of the following as a single vector.

4. \(\vec{m} + \vec{n}\)

5. \(\vec{n} - \vec{m}\)
9-6 Practice

Vectors

Write the component form of each vector.

1. \[ \vec{v} \]

2. \[ \vec{w} \]

Find the magnitude and direction of \( \vec{FG} \) for the given coordinates. Round to the nearest tenth.

3. \( F(-8, -5), G(-2, 7) \)

4. \( F(-4, 1), G(5, -6) \)

Graph the image of each figure under a translation by the given vector(s).

5. \( \triangle QRT \) with vertices \( Q(-1, 1), R(1, 4), T(5, 1); \vec{s} = (-2, -5) \)

6. trapezoid with vertices \( J(-4, -1), K(0, -1), L(-1, -3), M(-2, -3); \vec{c} = (5, 4) \) and \( \vec{d} = (-2, 1) \)

Find the magnitude and direction of each resultant for the given vectors.

7. \( \vec{a} = (-6, 4), \vec{b} = (4, 6) \)

8. \( \vec{c} = (-4, -5), \vec{f} = (-1, 3) \)

AVIATION For Exercises 9 and 10, use the following information.

A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.

9. Find the resultant velocity of the plane.

10. Find the resultant direction of the plane.
9-7  
Study Guide and Intervention  
Transformations with Matrices

Translations and Dilations  A vector can be represented by the ordered pair \( (x, y) \) or by the column matrix \( \begin{bmatrix} x \\ y \end{bmatrix} \). When the ordered pairs for all the vertices of a polygon are placed together, the resulting matrix is called the vertex matrix for the polygon.

For \( \triangle ABC \) with \( A(-2, 2), B(2, 1), \) and \( C(-1, -1) \), the vertex matrix for the triangle is \( \begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \).

**Example 1**  For \( \triangle ABC \) above, use a matrix to find the coordinates of the vertices of the image of \( \triangle ABC \) under the translation \( (x, y) \rightarrow (x + 3, y - 1) \).

To translate the figure 3 units to the right, add 3 to each x-coordinate. To translate the figure 1 unit down, add -1 to each y-coordinate.

<table>
<thead>
<tr>
<th>Vertex Matrix of ( \triangle ABC )</th>
<th>Translation Matrix of ( \triangle A'B'C' )</th>
<th>Vertex Matrix of ( \triangle A'B'C' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \begin{bmatrix} -2 &amp; 2 &amp; -1 \ 2 &amp; 1 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 3 &amp; 3 &amp; 3 \ -1 &amp; -1 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 5 &amp; 2 \ 1 &amp; 0 &amp; -2 \end{bmatrix} ]</td>
</tr>
</tbody>
</table>

The coordinates are \( A'(1, 1), B'(5, 0), \) and \( C'(2, -2) \).

**Example 2**  For \( \triangle ABC \) above, use a matrix to find the coordinates of the vertices of the image of \( \triangle ABC \) for a dilation centered at the origin with scale factor 3.

<table>
<thead>
<tr>
<th>Scale Factor of ( \triangle ABC )</th>
<th>Vertex Matrix of ( \triangle ABC )</th>
<th>Vertex Matrix of ( \triangle A'B'C' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 \times [ \begin{bmatrix} -2 &amp; 2 &amp; -1 \ 2 &amp; 1 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} -6 &amp; 6 &amp; -3 \ 6 &amp; 3 &amp; -3 \end{bmatrix} ]</td>
<td></td>
</tr>
</tbody>
</table>

The coordinates are \( A'(-6, 6), B'(6, 3), \) and \( C'(-3, -3) \).

**Exercises**

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations or dilations.

1. \( \triangle ABC \) with \( A(3, 1), B(-2, 4), C(-2, -1) \); \( (x, y) \rightarrow (x - 1, y + 2) \)

2. parallelogram \( RSTU \) with \( R(-4, -2), S(-3, 1), T(3, 4), U(2, 1) \); \( (x, y) \rightarrow (x - 4, y - 3) \)

3. rectangle \( PQRS \) with \( P(4, 0), Q(3, -3), R(-3, -1), S(-2, 2) \); \( (x, y) \rightarrow (x - 2, y + 1) \)

4. \( \triangle ABC \) with \( A(-2, -1), B(-2, -3), C(2, -1) \); dilation centered at the origin with scale factor 2

5. parallelogram \( RSTU \) with \( R(4, -2), S(-4, -1), T(-2, 3), U(6, 2) \); dilation centered at the origin with scale factor 1.5
Study Guide and Intervention (continued)

Transformations with Matrices

Reflections and Rotations When you reflect an image, one way to find the coordinates of the reflected vertices is to multiply the vertex matrix of the object by a reflection matrix. To perform more than one reflection, multiply by one reflection matrix to find the first image. Then multiply by the second matrix to find the final image. The matrices for reflections in the axes, the origin, and the line $y = x$ are shown below.

<table>
<thead>
<tr>
<th>For a reflection in the:</th>
<th>$x$-axis</th>
<th>$y$-axis</th>
<th>origin</th>
<th>line $y = x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply the vertex</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Example

$\triangle ABC$ has vertices $A(-2, 3), B(1, 4),$ and $C(3, 0).$ Use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ after a reflection in the $x$-axis.

To reflect in the $x$-axis, multiply the vertex matrix of $\triangle ABC$ by the reflection matrix for the $x$-axis.

Reflection Matrix for $x$-axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Vertex Matrix of $\triangle ABC$: $\begin{bmatrix} -2 & 1 & 3 \\ 3 & 4 & 0 \end{bmatrix}$

Vertex Matrix of $\triangle A'B'C'$: $\begin{bmatrix} -2 & 1 & 3 \\ -3 & -4 & 0 \end{bmatrix}$

Exercises

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

1. $\triangle ABC$ with $A(-3, 2), B(-1, 3), C(1, 0);$ reflection in the $x$-axis

2. $\triangle XYZ$ with $X(2, -1), Y(4, -3), Z(-2, 1);$ reflection in the $y$-axis

3. $\triangle ABC$ with $A(3, 4), B(-1, 0), C(-2, 4);$ reflection in the origin

4. parallelogram $RSTU$ with $R(-3, 2), S(3, 2), T(5, -1), U(-1, -1);$ reflection in the line $y = x$

5. $\triangle ABC$ with $A(2, 3), B(-1, 2), C(1, -1);$ reflection in the origin, then reflection in the line $y = x$

6. parallelogram $RSTU$ with $R(0, 2), S(4, 2), T(3, -2), U(-1, -2);$ reflection in the $x$-axis, then reflection in the $y$-axis
9-7 Practice
Transformations with Matrices

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations.

1. $\triangle KLM$ with $K(-7, -3)$, $L(4, 9)$, and $M(9, -6)$; $(x, y) \rightarrow (x - 7, y + 2)$

2. $\square ABCD$ with $A(-4, 3)$, $B(-2, 8)$, $C(3, 10)$, and $D(1, 5)$; $(x, y) \rightarrow (x + 3, y - 9)$

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.

3. quadrilateral $HIJK$ with $H(-2, 3)$, $I(2, 6)$, $J(8, 3)$, and $K(3, -4)$; $r = -\frac{1}{3}$

4. pentagon $DEFGH$ with $D(-8, -4)$, $E(-8, 2)$, $F(2, 6)$, $G(8, 0)$, and $H(4, -6)$; $r = \frac{5}{4}$

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

5. $\triangle QRS$ with $Q(-5, -4)$, $R(-1, -1)$, and $S(2, -6)$; $x$-axis

6. quadrilateral $WXYZ$ with $W(-4, -2)$, $X(-3, 4)$, $Y(2, 1)$, and $Z(4, -3)$; $y = x$

Use a matrix to find the coordinates of the vertices of the image of each figure under the given rotation.

7. $\square EFGH$ with $E(-5, -4)$, $F(-3, -1)$, $G(5, -1)$, and $H(3, -4)$; $90^\circ$ counterclockwise

8. quadrilateral $PSTU$ with $P(-3, 5)$, $S(2, 6)$, $T(8, 1)$, and $U(-6, -4)$; $270^\circ$ counterclockwise

9. FORESTRY A research botanist mapped a section of forested land on a coordinate grid to keep track of endangered plants in the region. The vertices of the map are $A(-2, 6)$, $B(9, 8)$, $C(14, 4)$, and $D(1, -1)$. After a month, the botanist has decided to decrease the research area to \(\frac{3}{4}\) of its original size. If the center for the reduction is $O(0, 0)$, what are the coordinates of the new research area?