Chapter 10 - Conic Sections - Chapter 10 Practice Test

Find the midpoint of the line segment with endpoints at the given coordinates.

1. \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{7 + (-5)}{2}, \frac{1 + 9}{2} \right) \]
   \[ = \left( \frac{2}{2}, \frac{10}{2} \right) \]
   \[ = (1, 5) \]

2. \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + (-8)}{8}, \frac{-1 + 2}{2} \right) \]
   \[ = \left( \frac{-49}{40}, \frac{1}{2} \right) \]
   \[ = \left( \frac{-49}{80}, \frac{1}{2} \right) \]

3. \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-13 + (-1)}{2}, \frac{0 + (-8)}{2} \right) \]
   \[ = \left( \frac{-14}{2}, \frac{-8}{2} \right) \]
   \[ = (-7, -4) \]

Find the distance between each pair of points with the given coordinates.

4. \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
   \[ = \sqrt{[3 - (-6)]^2 + (2 - 7)^2} \]
   \[ = \sqrt{9^2 + (-5)^2} \]
   \[ = \sqrt{81 + 25} \]
   \[ = \sqrt{106} \]

The distance between \((-6, 7)\) and \((3, 2)\) is \(\sqrt{106}\) units.
5. \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{\left( -\frac{3}{4} - \frac{1}{2} \right)^2 + \left( \frac{11}{4} - \frac{5}{2} \right)^2} \]
\[ = \sqrt{\left( \frac{5}{4} \right)^2 + \left( -\frac{21}{4} \right)^2} \]
\[ = \frac{25}{4} + \frac{441}{16} \]
\[ = \frac{466}{4} \]

The distance between \( \left( \frac{1}{2}, \frac{5}{2} \right) \) and \( \left( -\frac{3}{4}, -\frac{11}{4} \right) \) is \( \frac{466}{4} \) units.

6. \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(8 - 8)^2 + [-9 - (-1)]^2} \]
\[ = \sqrt{0^2 + (-8)^2} \]
\[ = \sqrt{64} \]
\[ = 8 \]

The distance between \( (8, -1) \) and \( (8, -9) \) is 8 units.

State whether the graph of each equation is a **parabola**, **circle**, **ellipse**, or **hyperbola**. Then graph the equation.

7. The equation is of the form \( Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \), where \( A = 1, C = 4, \) and \( F = -25 \). Since \( A \) and \( C \) have the same sign and \( A \neq C \), the graph is an ellipse.

8. The equation is of the form \( Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \), where \( A = 1, C = 1, \) and \( F = -36 \). Since \( A \) and \( C \) have the same sign and \( A = C \), the graph is a circle.
9. The equation is of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = 4, C = -26,$ and $F = 10$. Since $A$ and $C$ have opposite signs, the graph is a hyperbola.

![Hyperbola Diagram]

10. The equation is of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = -1, C = -1, D = -10,$ and $F = 24$. Since $A$ and $C$ have the same sign and $A = C$, the graph is a circle.

![Circle Diagram]

11. The equation is of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = \frac{1}{3}, E = -1,$ and $F = -4$. Since $C = 0$ and $A \neq 0$, the graph is a parabola.

![Parabola Diagram]

12. The equation is of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = -4, E = 1,$ and $F = -1$. Since $C = 0$ and $A \neq 0$, the graph is a parabola.

![Parabola Diagram]
13. \[(x + 4)^2 = 7(y + 5)\]
\[\frac{1}{7}(x + 4)^2 = y + 5\]
\[\frac{1}{7}(x + 4)^2 - 5 = y\]

The equation is of the form \(y = a(x - h)^2 + k\), so the graph is a parabola.

14. The equation is of the form \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\), where \(A = 25\), \(C = 49\), and \(F = -1225\). Since \(A\) and \(C\) have the same sign and \(A \neq C\), the graph is an ellipse.

15. The equation is of the form \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\), where \(A = 5\), \(C = -1\), and \(F = -49\). Since \(A\) and \(C\) have opposite signs, the graph is a hyperbola.

16. The equation is of the form \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\), where \(A = -\frac{1}{25}\), \(C = \frac{1}{9}\), and \(F = -1\). Since \(A\) and \(C\) have opposite signs, the graph is a hyperbola.
17. Write an equation to model the opening of the tunnel, assuming that the origin represents the midpoint of the width of the tunnel. Since the major axis is vertical, the equation is of the form \( \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \). The length of the major axis is 2(40) or 80 feet. Therefore, \( a = 40 \), so \( a^2 = 1600 \). The length of the minor axis is \( 2b = 60 \), or \( b = 30 \), so \( b^2 = 900 \). An equation of the ellipse is \( \frac{y^2}{1600} + \frac{x^2}{900} = 1 \). The height of the arch 12 feet from the edge of the tunnel is the value of \( y \) when \( x = \frac{1}{2}(60) - 12 \) or 18.

\[
\frac{y^2}{1600} + \frac{x^2}{900} = 1
\]

\[
\frac{y^2}{1600} + \frac{18^2}{900} = 1
\]

\[
\frac{y^2}{1600} = \frac{16}{25}
\]

\[
y^2 = 1024
\]

\[
y = 32
\]

The height of the arch 12 feet from the edge is 32 feet.

18. Find the exact solution(s) of each system of equations.

19. Substitute \( 2 - x \) for \( y \) in the first equation and solve for \( x \).

\( x^2 + y^2 = 100 \)
\( x^2 + (2 - x)^2 = 100 \)
\( x^2 + x^2 - 4x + 4 = 100 \)
\( 2x^2 - 4x + 4 = 100 \)
\( x^2 - 2x + 2 = 50 \)
\( x^2 - 2x - 48 = 0 \)
\( (x + 6)(x - 8) = 0 \)
\( x + 6 = 0 \) or \( x - 8 = 0 \)
\( x = -6 \) \( x = 8 \)

Now solve for \( y \).

\( y = 2 - x \)
\( y = 2 - (-6) \)
\( y = 2 - (8) \)
\( y = 8 \)
\( y = -6 \)

The solutions of the system are \((-6, 8)\) and \((8, -6)\).
20. \[ x + y = 1 \]
   \[ x = 1 - y \]

   Substitute \( 1 - y \) for \( x \) in the first equation and solve for \( y \).

   \[ x^2 + 2y^2 = 6 \]
   \[ (1 - y)^2 + 2y^2 = 6 \]
   \[ y^2 - 2y + 1 + 2y^2 = 0 \]
   \[ 3y^2 - 2y - 5 = 0 \]
   \[ (3y - 5)(y + 1) = 0 \]
   \[ 3y - 5 = 0 \quad \text{or} \quad y + 1 = 0 \]
   \[ 3y = 5 \quad \quad y = -1 \]
   \[ y = \frac{5}{3} \]

   Now solve for \( x \).

   \[ x = 1 - y \quad \quad x = 1 - y \]
   \[ x = 1 - \left( \frac{5}{3} \right) \quad x = 1 - (-1) \]
   \[ x = -\frac{2}{3} \quad \quad x = 2 \]

   The solutions of the system are \( \left( -\frac{2}{3}, \frac{5}{3} \right) \) and \( (2, -1) \).
21. Use elimination to solve the system.

\[
\begin{align*}
 x^2 - y^2 & -12x + 12y = 36 \\
 (+) x^2 + y^2 & -12x - 12y + 36 = 0 \\
 2x^2 & -24x + 36 = 36 \\
 2x^2 - 24x & = 0 \\
 x^2 - 12x & = 0 \\
 x(x - 12) & = 0 \\
 x = 0 & \text{ or } x - 12 = 0 \\
 x = 0 & \text{ or } x = 12 \\
 x & = 12
\end{align*}
\]

Now solve for y.

\[
\begin{align*}
 x^2 - y^2 - 12x + 12y & = 36 \\
 (0)^2 - y^2 - 12(0) + 12y & = 36 \\
 -y^2 + 12y & = 36 \\
 -y^2 + 12y - 36 & = 0 \\
 -(y - 6)^2 & = 0 \\
 y - 6 & = 0 \\
 y & = 6 \\
 x^2 - y^2 - 12x + 12y & = 36 \\
 (12)^2 - y^2 - 12(12) + 12y & = 36 \\
 -y^2 + 12y & = 36 \\
 -y^2 + 12y - 36 & = 0 \\
 -(y - 6)^2 & = 0 \\
 y - 6 & = 0 \\
 y & = 6 
\end{align*}
\]

The solutions of the system are (0, 6) and (12, 6).

**FORESTRY** Use the following information.

A forest ranger at an outpost in the Fishlake National Forest in Utah and another ranger at the primary station both heard an explosion. The outpost and the primary station are 6 kilometers apart.
22. The distance that sound travels in 6 seconds is 
   \((0.35 \text{ km/s}) \cdot (6 \text{ s}) = 2.1 \text{ km}\).
   The difference in distances from the explosion to the two rangers is constant = 2.1 km.
   Therefore the possible locations of the explosion can be represented by a hyperbola with the rangers at the foci
   and the difference in distances of the points on the hyperbola from the foci being 2.1 km. Let the equation of
   the hyperbola be 
   \[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]
   Here \(2a = 2.1\).
   So \(a = 1.05\) and 
   \(a^2 = 1.1025\).
   The distance between the stations is 6 km.
   So \(2c = 6\) and 
   \(c = 3\).
   Use \(b^2 = c^2 - a^2\) to find \(b^2\).
   \(b^2 = (3)^2 - (1.05)^2\)
   \(b^2 = 9 - 1.1025\)
   \(b^2 = 7.8975\)
   Substitute for \(a^2\) and \(b^2\) to get the equation. 
   \[ \frac{x^2}{1.1025} - \frac{y^2}{7.8975} = 1 \]

23. 

24. C; The equation is of the form \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\), where
   \(A = 1, C = 5, E = 8,\) and \(F = -8\). Since \(A\) and \(C\) have the same sign and \(A \neq C\), the graph is an ellipse.